



MATHEMATICS SAMPLE ENTRY EXAMINATION 2018

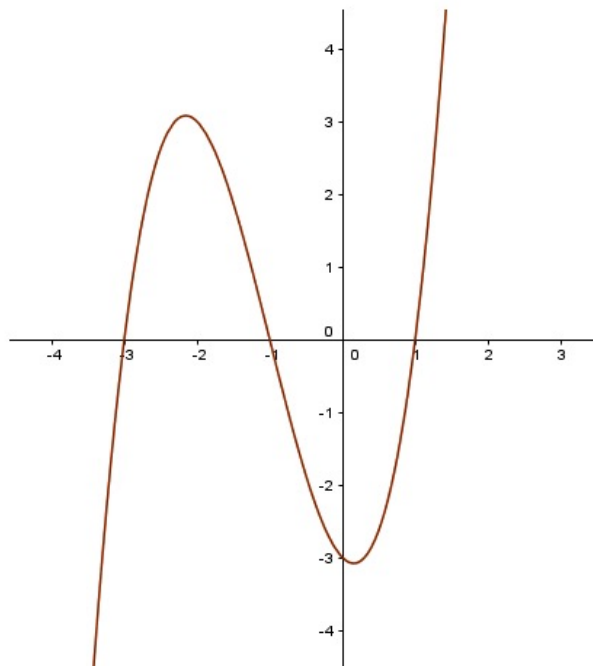
Examiners: Dr Sarah Tanner, Dr Siobhan Connolly Kernan & Dr Rajesh Jaiswal

Answer five questions in total, answer any two questions from Section A and answer any three questions from Section B.

Section A (200 marks) Answer any two questions from this section.**Section A, Question 1** Answer two questions from Section A

(100 marks)

- (a) Below is the graph of $f(x) = x^3 + 3x^2 - x - 3$. Use the graph to answer the following questions.



- i. Write down the roots of the equation $f(x) = 0$. (10 marks)
- ii. When $x = 0.5$ what is the corresponding y value? (10 marks)
- iii. What is $f(-2)$? (10 marks)
- iv. For what value(s) of x is $f(x) = 1$? (10 marks)
- v. For what values of x is the graph of $f(x)$ decreasing? (12 marks)
- vi. For what values of x is $f(x) > 0$? (12 marks)

- (b) The slope of the line through the points $(2, a)$ and $(7, 3a)$ is $\frac{3}{5}$. Find the value of a . (12 marks)
- (c) Find the equation of the line containing the point $(2, 3)$ that is perpendicular to the line $5x + 4y - 11 = 0$. (12 marks)
- (d) The point $(-3, b)$ is on the line $5x + 4y + 3 = 0$. Find the value of b . (12 marks)

Section A, Question 2 Answer two questions from Section A (100 marks)

(a) Solve the following for x : (25 marks)

$$x - 1 = \sqrt{3x - 5}.$$

(b) Express the following as a single fraction: (25 marks)

$$\frac{2}{3t-1} - \frac{5}{2t+2} \text{ where } t \neq \frac{1}{3} \text{ and } t \neq -1$$

(c) Solve the following pair equations simultaneously. (25 marks)

$$2x^2 - y^2 = 14$$

$$x - y = 1$$

(d) Factorise $16a^2 - (3a - b)^2$. (25 marks)

Section A, Question 3 Answer two questions from Section A

(100 marks)

- (a) Find the mean and the standard deviation of the data in the following frequency table.

(20 marks)

x	20	30	40	50
f	18	32	26	24

- (b) A urn containing 8 red balls and 2 blue balls. A ball is drawn at random from the bag and then replaced before a second ball is drawn.
- Find the probability that the first ball drawn is blue (20 marks)
 - Find the probability that the first ball drawn is blue and the second is red. (20 marks)
 - If the balls are drawn from the urn one at a time but are not replaced, find the probability that the first ball counter drawn is blue and the second is red. (20 marks)
- (c) Find the twentieth term and the sum of the first 20 terms of the arithmetic sequence whose first term is 3 and whose common difference is 4. (20 marks)

Section B (300 marks) Answer any three questions from this section.

Section B, Question 4 Answer three questions from Section B (100 marks)

(a) Differentiate the following functions with respect to x

i. $f(x) = -\frac{2}{3}x^{-1} + \frac{e^{-x}}{\sin(x)}$ (20 marks)

ii. $f(x) = (x^6 - \frac{1}{7}x^4) \sin x$ (20 marks)

iii. $f(x) = (2x^3 - 3x)^3$ (20 marks)

(b) Consider the function, $f(x) = x^3 - 3x^2 - 45x + 2$

i. Calculate the first and second derivatives, $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ (20 marks)

ii. Hence, give the (x, y) coordinates of any critical points. (15 marks)

iii. Using the calculations above, say whether these are local maxima or minima. (5 marks)

Section B, Question 5 Answer three questions from Section B (100 marks)

(a) Simplify the following using rules of logs as appropriate

i. $\log_{10}(5) + \log_{10}(7) + \log_{10}(2)$ (20 marks)

ii. $-2\log_3(3) + \log_3(7) - 2\log_6(5) - 2\log_6(5)$ (20 marks)

(b) Solve the following equation for x (20 marks)

$$\log_{10}(x - 1) + \log_{10}(x + 1) = 2$$

(c) Solve the following set of equations (20 marks)

$$3x + y = 6$$

$$x^2 + y^2 = 8$$

(d) Given the function, $f(x) = -\frac{2}{3}x - 4$ Calculate the inverse function, $f^{-1}(x)$, and, further, verify that $f(f^{-1}(x)) = x$ (20 marks)

Section B, Question 6 Answer three questions from Section B

(100 marks)

(a) Simplify the following complex numbers and write in standard form

i. $(8 - 6i)(8 + 6i)$ (20 marks)

ii. $\frac{(3-i)}{(2+7i)}$ (20 marks)

(b) Given the complex number, $z = -\sqrt{3} + i$, Use De Moivre's Theorem to write z^4 in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$ (20 marks)

(c) Calculate the value of following indefinite integral (20 marks)

$$\int (2x^{-5} + x^2 + 3)dx$$

(d) Find the area underneath the curve defined by the function $f(x) = 2 \sin(x) + 3x^2$ between $x = 0$ and $x = 2\pi$ (20 marks)

Section B, Question 7 Answer three questions from Section B (100 marks)

- (a) The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$. Find the value of x . (30 marks)
- (b) Given that $n \in \mathbb{N}$, prove by induction that 3 divides $4^n + 5$. (35 marks)
- (c) Given the following equations which represent two circles. Prove that both circles touch externally and find the point of contact (35 marks)

$$x^2 + y^2 + 2x - 2y - 23 = 0$$

$$x^2 + y^2 - 14x - 2y + 41 = 0$$